SIE 2015
6th International Symposium on Industrial Engineering
24th-25th September 2015
FACULTY OF MECHANICAL ENGINEERING
STEINBEIS ADVANCED RISK TECHNOLOGIES
INNOVATION CENTER FME

THE USE OF FUZZY LOGIC IN RISK MODELING OF INSURANCE COMPANIES OPERATIONS

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Abstract. *The significance of fuzzy logic is the possibility of risk management without quantitative models. Data, obtained using fuzzy logic, may allow policyholders the efficient process of making less risky decisions. Furthermore, they provide the apparatus for determining the risk of insurer's business, and based on that policyholders can decide whether to enter into a contract with that company exactly.*
Key words: *Risk, fuzzy logic, insurance.*

1. BASIC CONCEPTS OF FUZZY THEORY

The concept of fuzzy sets was introduced as a generalization of classical sets, forming a characteristic function - the membership function $\mu_A(x)$. References used in the preparation of this part of the work are [1,2,5] . In classical sets, the belonging of an element to some set A is precisely defined, therefore the characteristic function $\mu_A(x)$ can take only two values, 1 or 0

$$\mu_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}.$$

I
n fuzzy sets some element can partially belong to a set. If it is defined that the characteristic function takes values in the interval [0, 1], the membership concept is no longer ordinary, but it becomes fuzzy (in terms of partial membership).

Let A be a classical subset of the universal set X, which can be finite or infinite. Fuzzy set A_μ is defined as

$$A_\mu = \{ (x, \mu_A(x)) \mid x \in A, \mu_A(x) \in [0,1] \}$$

where $\mu_A(x)$ is a membership function and it represents the degree of belonging of some element from the set A to the fuzzy set A_μ, [2]. Larger value of $\mu_A(x)$ denote a higher degree of membership. Each fuzzy set is completely and in a unique way determined by its membership function.

Linguistic variables, such as very little, little, medium, and so on whose interpretation depend on a context, form the basis of fuzzy logic. Linguistic terms which represent approximate values of a base variable are presented with fuzzy numbers. Each linguistic variable is interpreted as a fuzzy number and it is defined in terms of a base variable, the values of which are real numbers. A linguistic variable takes the value from its set of terms which represents the set of labels to which fuzzy sets are assigned over the same domain, a fuzzy relation with the characteristic fuzzy function $\mu_A(x)$. Each linguistic value can be interpreted as a label of fuzzy subset of overall speech whose main variable is numerical value.

2. MODEL

Risk intensity assesment is a difficult and complicated process. In order to make the data comperable it is necessary to define a unique model of risk assessment. One of possible ways is that risk implies the possibility of occurence of adverse effects on insurance companies' business and financial results. In other words, it is necessary to determine the sufficiency of funds (premiums) for resolving of caused adverse events. The model given in this study is based on fuzzy environment used to determine if an insurance company conducts risky business, on the basis of which future policyholders can make a decision whether they want to enter into a contract (policy) with that insurer exactly.

The main characteristic of fuzzy sets is the fact that a membership function is not strictly defined. Its formulation and input parameters depend on expert knowledge. Fuzzy logic can be managed by risks that don't have a quantitative model. Using them it is possible to assess the degree of risk exposure and explain dependence and relations among factors.

The risk intensity may be defined as numeric variable which takes values in the interval [o, *x*], or as linguistic variable which is defined by terms "low, medium, high etc".

Based on expert knowledge the risk intensity, as numeric variable, takes value in the interval [0, 100], Figure 1.

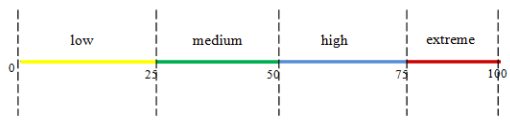


Figure 1 Classic illustration of risk intensity

Fuzzy system consists of two input linguistic variables (the total amount of premium and the amount of resolved claims) and the level of risk tolerance (intensity), as one output variable. One of the main indicators of insurance company's operation is a premium (as one of the main sources of income) and payment of damage claims (one of the main sources of expenditure). Based on these parameters it is possible to determine the risks of the insurance company's operations, that is the risk tolerance assessment, which is very important to a future policyholder (a decision-maker) when deciding whether he will do the business with that company. The references used in the process of forming the model are [3, 4, 6, 7, 8].

The model is applied to the 01-Accident Insurance. For the total amount of premium – TAP as well as for the amount of resolved claims – ARC, the average value of the amount of premium and the amount of claims is taken, for the insurance type 01, over the last 5 years in some insurance company. The input values are given numerically and for the total amount of premium is obtained TAP=73, and for the amount of resolved claims ARC=61 tens of millions RSD.

The next step is the fuzzification of input variables into linguistic variables which take values from the set of terms {S-*low* (*small*), M-*medium*, H-*high* and E-*extreme*}. It is determined that input variables each have three linguistic values while output variables have four of them, therefore their corresponding sets of conditions are as follows:

TAP (*the total amount of premium*) = {S, M ,H},
ARC (*the smount of resolved claims*)={S, M, H},
RI (*risk intensity*) = {S, M, H, E}.

Larger number of linguistic values is not required because the accuracy of the output is acquired. The corresponding confidence intervals of linguistic variables are given as follows:

$$I_1 = \{x_1 \times 10^7 | 0 \le x_1 \le 100\},$$
$$I_2 = \{x_2 \times 10^7 | 0 \le x_2 \le 100\}$$
$$I_3 = \{r | 0 \le r \le 100\}.$$

Real numbers x_1 and x_2 represent consecutively the total amount of a premium and the amount of resolved claims in tens of millions dinars, while *r* is the variable which is in the interval [0, 100] and it represents a scale measuring the risk intensity.

Each linguistic variable, with fuzzy numbers as argumnets, is determined by linguistic values, which are determined by membership functions. Evaluations of input linguistic variables are given by triangular and trapezoidal membership functions. Their shapes are shown in Figure 2 and analytically described in the following way:

$$\mu_m(x) = \begin{cases} 1, & 0 \le x \le 25 \\ \dfrac{50-x}{25}, & 25 \le x \le 50 \\ 0, & x \ge 50 \end{cases}$$

$$\mu_s(x) = \begin{cases} 0, & 0 \le x \le 25 \\ \dfrac{x-25}{25}, & 25 \le x \le 50 \\ \dfrac{75-x}{25}, & 50 \le x \le 75 \\ 0, & x \ge 75 \end{cases}$$

$$\mu_v(x) = \begin{cases} \dfrac{x-50}{25}, & 50 \le x \le 75 \\ 1, & 75 \le x \le 100 \end{cases}.$$

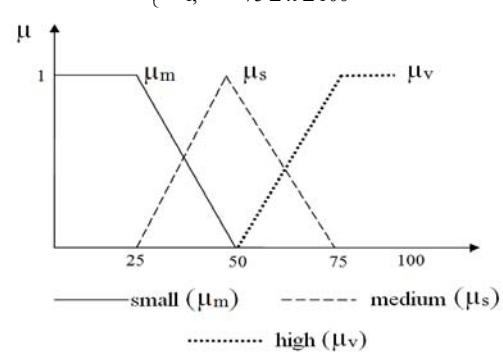


Figure 2 Graphic illustration of membership function of input variables

The corresponding fuzzy input values are:

$$\mu_s(73) = \frac{2}{25}, \ \mu_v(73) = \frac{23}{25}$$
$$\mu_s(61) = \frac{14}{25}, \ \mu_v(61) = \frac{11}{25}.$$

They are obtained by coding of input values, or by replacing initial values with the corresponding membership function. The transformation of initial values of input variables TAP=73 and ARC=61 (real numbers) into corresponding values of linguistic variables (fuzzy input values) is shown graphically in Figures 3 and 4. The straight line x=73 (x=61) intersects only fuzzy numbers S and H, therefore the cross-section of the straight line x=73 (x=61) and fuzzy number S is empty and the value of the corresponding membership function μ_m, 0. Non-empty cross-sections of the straight line x=73 (x=61) and two remaining fuzzy numbers represent the fuzzy shape of input value $\mu_s(73)$, $\mu_v(73)$, $\mu_s(61)$ i $\mu_v(61)$, (see [2]).

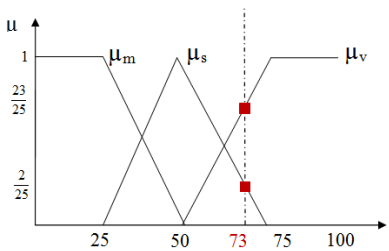


Figure 3 Fuzzy input value for the amount of the collected premium TAP=73

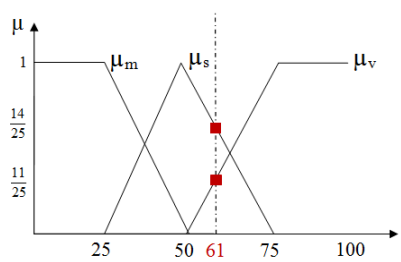


Figure 4 Fuzzy input value for the amount of resolved claims ARC=61

Models based on fuzzy logic consists of if-then rules [2, 8]. The number of rules is obtained as the multiplication of the number of input linguistic variables values and this model has 9 of them. After defining input fuzzy numbers and the application of 9 if-then rules, a decision table is obtained [2].

Table 1 The decision table

	S	**M**	**H**
S	S	S	M
M	S	M	H
H	M	H	E

In order that obtained measured fuzzy input values are applicable, the table 1 needs to be reduced. In this model the reducing is carried out using Mamdani's if-then rules, that are based on a minimum operator [1]. Their application results in the cause-effect table.

Table 2 The cause-effect table

	$\mu_s(61)$	$\mu_v(61)$	0
$\mu_s(73)$	$\mu_s(r)$	$\mu_m(r)$	0
$\mu_v(73)$	$\mu_m(r)$	$\mu_e(r)$	0
0	0	0	0

In fuzzy logic, the logical operator ∧ is being replaced by taking the minimum value under certain conditions, that are "min" operations [4], and the resluts obtained by its application are shown in the following table.

Table 3. Results

	$\mu_s(61)=\frac{14}{25}$	$\mu_v(61)=\frac{11}{25}$	0
$\mu_s(73)=\frac{2}{25}$	$\min\left(\frac{2}{25},\frac{14}{25}\right)=\frac{2}{25}$	$\min\left(\frac{2}{25},\frac{11}{25}\right)=\frac{2}{25}$	0
$\mu_v(73)=\frac{23}{25}$	$\min\left(\frac{14}{25},\frac{23}{25}\right)=\frac{14}{25}$	$\min\left(\frac{11}{25},\frac{23}{25}\right)=\frac{11}{25}$	0
0	0	0	0

This clearly shows that four active cells are available for the decision table for the risk intensity assessment model to the decision maker.
By taking the minimum value of corresponding fields from tables 2 and 3, and by applying the operator ∧, the output control variables are obtained, which are shown in table 4. It should be noted that in this model the minimum operator is applied to the number and the membership function of the fuzzy number.

Table 4 The table of output control variables

$\frac{2}{25}\wedge\mu_s(r)=\min\left(\frac{2}{25},\mu_s(r)\right)$	$\frac{2}{25}\wedge\mu_v(r)=\min\left(\frac{2}{25},\mu_m(r)\right)$
$\frac{14}{25}\wedge\mu_v(r)=\min\left(\frac{14}{25},\mu_m(r)\right)$	$\frac{11}{25}\wedge\mu_e(r)=\min\left(\frac{11}{25},\mu_e(r)\right)$

As a result four output fuzzy numbers are obtained, and it is necessary to get one fuzzy output variable with the membership function (aggregation process). In other words, it is necessary to make a definite conclusion based on a series of separate conclusions. The aggregation process itself is shown graphically and it follows [2].

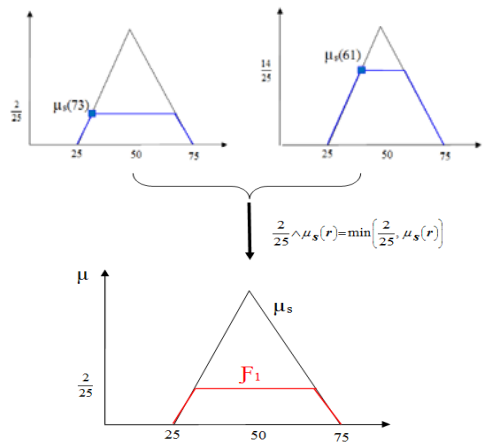
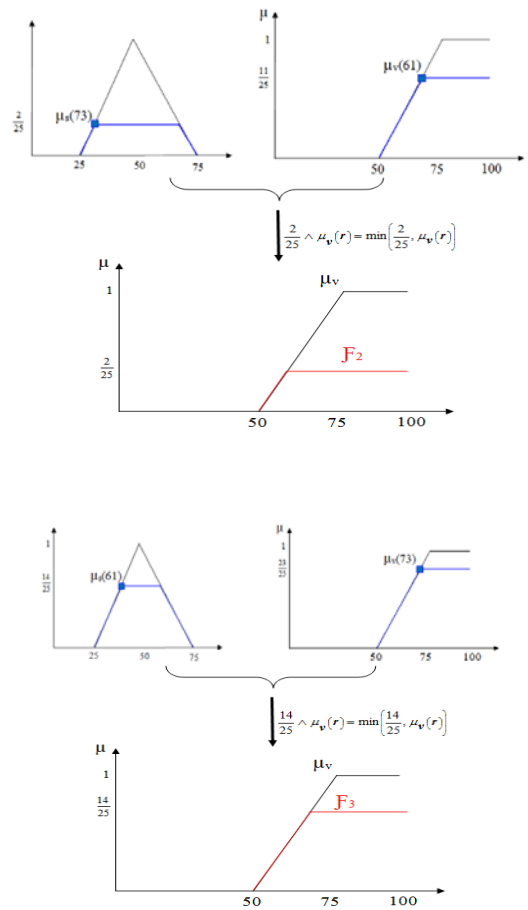


Figure 5a. The graphic illustration of the aggregation process



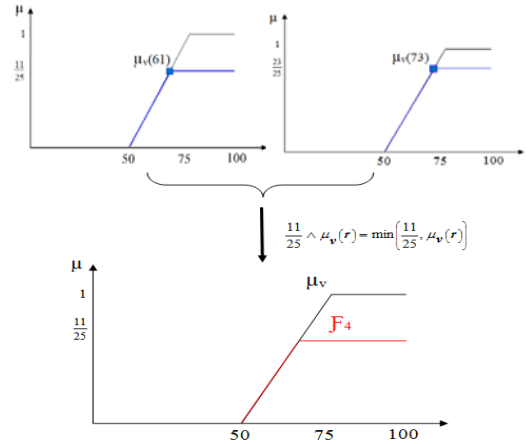


Figure 5b. The graphic illustration of the aggregation process

Output trapezoidal fuzzy numbers F_1, F_2, F_3 and F_4 are obtained and they need to be shown as one output fuzzy number. In geometric sense, it is necessary to find suprema of trapezoids obtained. In Figure 5 it is noticed that output values obtained by rules 2 and 4 are contained in the output value from the rule number 3. Regarding that, it is sufficient to sublimate output values obtained by rules 1 and 3 using a maximum operator and get the aggregate function $\mu_{agg}(r)$. Or

$$\mu_{agg}(r) = \max\left(\min\left(\frac{2}{25}, \mu_s(r)\right), \min\left(\frac{14}{25}, \mu_v(r)\right)\right)$$

whose graph is given in Figure 6.

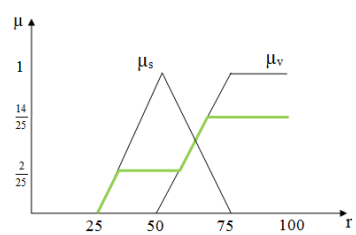


Figure 6 Fuzzy output value after the application of the aggregation

The aggregate functional form is:

$$\mu_{agg}(r) = \begin{cases} \dfrac{r-20}{25}, & 25 \le r \le 27 \\ \dfrac{2}{25}, & 27 \le r \le 52 \\ \dfrac{r-50}{25}, & 52 \le r \le 64 \\ \dfrac{14}{25}, & 64 \le r \le 100 \end{cases}.$$

As the result of aggregation, a fuzzy set as a cross-section of fuzzy sets is obtained and it is hardly applicable in that form, so it is necessary to carry out a defuzzification of the result.

The resulting fuzzy output variable needs to be converted into real value, that is the defuzzification process should be applied by using Center of area method-CAM method (see [2]). The method is based on a weighted average and it is the most applicable in practice. A domain aggregate function [25, 100] is divided into 5 subintervals of equal length-15. Replacing $r_i = 40$, 55, 70 and 85 with membership function of output fuzzy number results in the values in the following order $\frac{2}{25}, \frac{1}{5}, \frac{14}{25}, \frac{14}{25}$.

The required value is:

$$r = \frac{40 \cdot \frac{2}{25} + 55 \cdot \frac{1}{5} + 70 \cdot \frac{14}{25} + 85 \cdot \frac{14}{25}}{\frac{2}{25} + \frac{1}{5} + \frac{14}{25} + \frac{14}{25}} = 34 .$$

Based on these results follows that on the scale of 0-100, the level of risk is 34.

3. CONCLUSION

Based on the experience of financial experts the risk intensity scale is formed, Figure 1, in view of that can be concluded that the future investment is a medium risk investment. That means that insurers' business is not considered to be risky and the future policyholder can enter into a contract or a policy of Accident Insurance with the company relying on given level of risk, which refers to a sufficiency of collected premiums that enables the insurer to have funds for claims payment.

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