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OPTIMAL PRICING STRATEGIES FOR INTERNET MARKETING ON TIME-SENSITIVE PRODUCTS

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Abstract. *Internet marketing is a form of marketing strategy which utilizes the internet always to promote business by providing discounted selling price. Since the customers' demand depends on the selling price, how to determine the pricing strategy is the priority to the retailer. In this study, a profit model is developed and the optimal solution of the model is derived. An algorithm is developed to obtain a pricing strategy such that the profit is maximized. Numerical examples and sensitivity analysis are presented to illustrate the model.*
Key words: *Internet marketing; Time-sensitive product; Pricing*

1.INTRODUCTION

Internet marketing is a form of marketing strategy which utilizes the internet to promote business. Enterprises can deliver their products, services and advertising through their commercial websites. Consumers acquire information and purchase products directly from these websites. Nowadays, most enterprises have been practicing internet marketing and advertising time-sensitive products. Time-sensitive product is heavily advertised for a period of time and loses its original value after the sell-by date; examples include movie tickets, train and plane tickets. A time-sensitive product is different from other traditional commodities, as it bears potentially higher loss after the sales period.

For example, as a result of globalization in recent decades, the demand for air transportation has increased, and with increasing accessibility to the internet, online travel companies servicing hotels and airfares were created. These websites purchase flight tickets from the airline companies, and in turn post these flight tickets on their own websites. Online travel websites usually offer cheaper fares in order to promote online bookings and increase demand. Customers can search and compare airfare information among websites before placing their orders. Since, the availability of airplane seats is time-sensitive, websites companies gradually increase ticket prices on the websites as the departure date is approaching, and decrease ticket prices a few days before the departure to sell out the remaining tickets. This study aims to determine what pricing strategy is most effective and to determine how many seat tickets to purchase, in order to maximize profit and reach a win-win scenario for both the website company and customer.

Kimes et al. (1998) classified the time-sensitive products by price and demand, implemented diverse pricing options for different products and predicted market demand. Shih et. al. (2013) developed a search engine optimization (SEO) mechanism for internet marketing strategy that can be used by an enterprise to improve the ranking of its website in the search engine results. Most research in the past discussed the concept and technology of internet marketing, but little on the inventory problem. This study aims to determine what pricing strategy is most effective and to determine the optimal ordering quantities to reach a win-win scenario for both the website and customer.

2. ASSUMPTIONS AND NOTATION

The mathematical models presented in this study have the following assumptions:

(1). There are no interdependencies between ordered items, and therefore a single item model is assumed.
(2). The demand rate is deterministic and stationary through time.
(3). The replenishment is instantaneous.
(4). No shortages are allowed.

Meanwhile, the mathematical models have the following notations:

T	The selling period
t_1	the critical time of the largest demand during selling period; for considering real situation, $t_1 > T/2$
c_p	the unit purchase cost, \$/unit
K	the maximal unit selling price, \$/unit
Δ	discount rate for market price; market price: $c_p + \Delta(K - c_p)$
$p(t)$	the unit discounted selling price function of time t , \$/unit, $c_p < p(t) < K$
$D(t)$	The real demand function of time t
F	constant, used in Eq.(5) and Eq.(6)
G	constant, used in Eq.(5) and Eq.(6)
Q	the ordering quantity
c_o	the ordering cost, \$/order
δ_1	lower discount rate, that is 100(1- δ_1)% off, decision variable
δ_2	higher discount rate, that is 100(1- δ_2)% off, $0 < \delta_1 < \delta_2 < 1$., decision variable
h	inventory holding cost per item, \$/unit/unit time
TR	the total revenue per cycle
TC	the total cost per cycle
TPU	the net profit per unit time

3. ANALYSIS OF THE MODEL

In this section, a model is formulated to obtain the net profit. Throughout this study, a single product is assumed. The retailer orders a batch from the supplier of the products, Q, with the unit purchase cost c_p, and sells to customers with discounted selling price $p(t)$ on the internet for promotion sake. Since the customers' demand depends on the selling price $p(t)$, it is important for the retailer to know how to price the item for the optimal profit. Assume that the items (eg. airplane seats) are time-sensitive, generally, the customers' imaginary demand (the demand that customers expect to get them without considering the price) increases during time [0, t_1] while it decreases during time [t_1, T] (Please refers to Fig. 1). Due to this situation, the retailers gradually increase the prices of the items as the critical time t_1 is approaching, and decrease its prices a few days before the end of selling period T. (Please refer to the unit market selling price in Fig.2) Thus, the real demand will be affected by the price. In this study, if the retailer improves management and administration of internet transaction, then the discounted selling price $p(t)$ will be used for promotion sake. (Please refer to the discounted selling price in Fig.2)

The unit discounted selling price function $p(t)$ is as follows,

$$p(t) = \begin{cases} p_1(t), 0 \le t < t_1, \\ p_2(t), t_1 \le t < T. \end{cases} \quad (1)$$

Where

$$p_1(t) = c_p + \delta_1(K - c_p) + \frac{(\delta_2 - \delta_1)(K - c_p)}{t_1} t, 0 < \delta_1 < \delta_2 < 1, 0 < t < t_1. \quad (2)$$

$$p_2(t) = c_p + \delta_1(K - c_p) + \frac{(\delta_2 - \delta_1)(K - c_p)}{t_1 - T}(t - T), 0 < \delta_1 < \delta_2 < 1, t_1 < t < T. \quad (3)$$

Responding to the unit discounted selling price, the real demand $D(t)$ is as follows,

$$D(t) = \begin{cases} D_1(t), 0 \le t < t_1, \\ D_2(t), t_1 \le t < T. \end{cases} \quad (4)$$

Where

$$D_1(t) = \frac{F - p_1(t)}{G}, 0 \le t < t_1, \quad (5)$$

$$D_2(t) = \frac{F - p_2(t)}{G}, t_1 \le t < T. \quad (6)$$

With F, G are constants, $F > K$.

From the above assumptions and notation, we know that the inventory level $I(t)$ at time t satisfies the following two differential equations:

$$dI(t)/dt = -D_2(t), \quad t_1 \le t \le T. \quad (7)$$

with initial condition $I(T) = 0$, one has

$$I(t) = \frac{(T-t)(2FT - 2c_pT + \delta_2c_pT - \delta_2KT + \delta_1c_pT - \delta_1KT + \delta_2Kt - \delta_2c_pt + 2c_pt_1 - 2Ft_1 + 2\delta_1Kt_1 - 2\delta_1c_pt_1 + \delta_1c_pT - \delta_1KT)}{2(T-t_1)G}, t_1 \le t \le T. \quad (8)$$

Then $I(t_1) = \dfrac{(2FT - 2c_pT + \delta_2c_pT - \delta_2KT + \delta_1c_pT - \delta_1KT + \delta_2Kt - \delta_2c_pt + 2c_pt_1 - 2Ft_1 + 2\delta_1Kt_1 - 2\delta_1c_pt_1 + \delta_1c_pT - \delta_1KT)}{2G}$. (9)

And $dI(t)/dt = -D_1(t)$, $0 \le t < t_1$, (10)

with initial condition $\lim_{t \to t_1^+} I(t) = I(t_1)$, one has

$$I(t) = \frac{(\delta_1 - \delta_2)(c_p - K)t^2 - 2t_1(F - c_p - \delta_1K + \delta_1c_p)t + t_1T(2F - 2c_p + \delta_2c_p - \delta_2K + \delta_1c_p - \delta_1K)}{2t_1G}, \quad 0 \le t < t_1, \quad (11)$$

With the ordering quantity Q,

$$Q = I(0) = \frac{T(2F - 2c_p + \delta_2c_p - \delta_2K + \delta_1c_p - \delta_1K)}{2G}, \quad (12)$$

The inventory system is shown in Fig.4.

The total revenue per cycle $TR(\delta_1, \delta_2)$ is as follows,

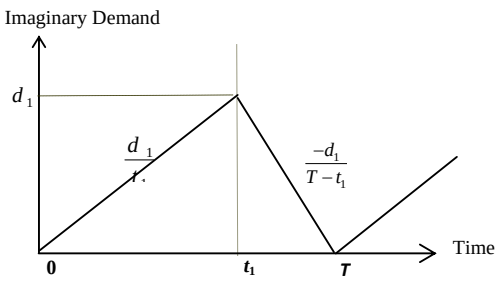

Figure 1. The figure of customers' imaginary demand.

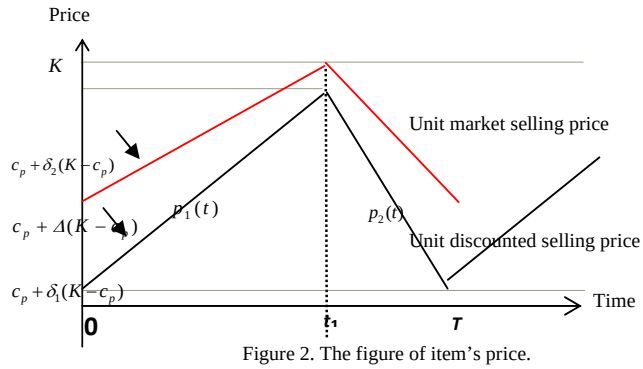

Figure 2. The figure of item's price.

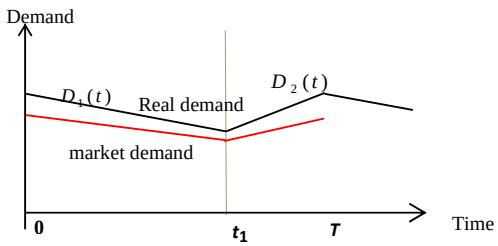

Figure 3. The figure of customers' real demand.

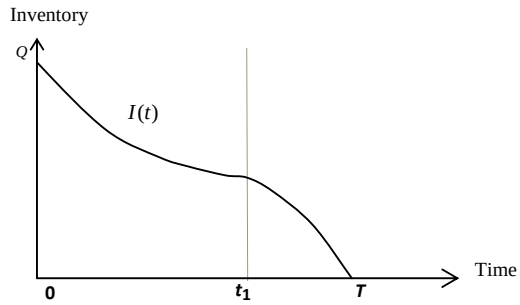

Figure 4. The figure of inventory system.

$$TR(\delta_1,\delta_2)=\int_0^T D(t)\,p(t)dt=[2Tc_p^2(3\delta_1+3\delta_2-\delta_1^2-\delta_2^2-\delta_1\delta_2-3)-2TK^2(\delta_1^2+\delta_2^2+\delta_1\delta_2)-T(6\delta_1Kc_p+6\delta_2Kc_p$$
$$+3\delta_1Fc_p+3\delta_2Fc_p-4\delta_1^2Kc_p-4\delta_2^2Kc_p-6Fc_p-3\delta_1KF-3\delta_2KF-4\delta_1\delta_2Kc_p)]/(6G) \quad (13)$$

The total cost per cycle $TC(\delta_1,\delta_2)$ is as follows,

$$TC(\delta_1,\delta_2)=\text{Purchase cost + Inventory cost + Ordering cost}=QC_p+\int_0^T I(t)dt\ h+C_o, \quad (14)$$

where

$$\int_0^T I(t)dt=[T^2(\delta_2c_p-\delta_2K+2\delta_1c_p-2\delta_1K+3F-3c_p)-t_1T(\delta_2-\delta_1)(K-c_p)]/(6G) \quad (15)$$

The net profit per unit time $TPU(\delta_1,\delta_2)$ is as follows,

$$TPU(\delta_1,\delta_2)=\frac{1}{T}[\,TR(\delta_1,\delta_2)\text{-}TC(\delta_1,\delta_2)] \quad (16)$$

Since the unit discounted selling price $p(t)$ of the item is lower than that of market price $c_p+\Delta(K-c_p)$ for marketing, then the problem can be formulated as follows:

Maximize: $TPU(\delta_1,\delta_2)$ (17)

Subject to: $0\le\delta_1\le\delta_2\le1,\ 0\le\delta_1\le\Delta.$

From Eq.(17), the domain of the problem is closed and bounded, which means the optimum of the problem occurs at either relative maximum of $TPU(\delta_1,\delta_2)$ in the interior of the domain or at the boundary of the domain, $\delta_1=0,\delta_1=\Delta,\delta_1=\delta_2,\delta_2=1.$

The following solution procedure is used

Solution procedure

Step 1. Check the concavity of $TPU(\delta_1,\delta_2)$.(Hessian matrix function of $TPU(\delta_1,\delta_2)$ is positive)

Step 2. Find both the relative maximum of $TPU(\delta_1,\delta_2)$ in the interior of the domain and at the boundary of the domain.

Step 3. Find the maximal value of Step 2, the optimum is obtained.

Since

$$\partial^2TPU\big/\partial\delta_1^2=\frac{-2(c_p-K)^2}{3G}<0, \quad (18)$$

$$\partial^2TPU\big/\partial\delta_2^2=\frac{-2(c_p-K)^2}{3G}<0, \quad (19)$$

$$\partial^2TPU\big/\partial\delta_1\partial\delta_2=\frac{(c_p-K)^2}{3G}, \quad (20)$$

and

$$\text{Hessian}(\delta_1,\delta_2)=\frac{(c_p-K)^4}{3G^2}>0, \quad (21)$$

which leads to the function $TPU(\delta_1, \delta_2)$ being strictly convex with respect to (δ_1, δ_2). The positive-definite Hessian matrix results in optimal (δ_1^*, δ_2^*) values without restriction. Hence, setting $\partial TPU/\partial\delta_1 = 0$, and $\partial TPU/\partial\delta_2 = 0$, the optimal discount rate δ_1^*, δ_2^* without restriction, can be derived by MAPLE 13 software as follows:

$\delta_1 = \dfrac{F - c_p + Th - t_1 h}{2(K - c_p)}$, and the optimal higher discount rate $\delta_1^* = \min\{\delta_1, 1\}$ (22)

$\delta_2 = \dfrac{F - c_p + t_1 h}{2(K - c_p)}$, and the optimal lower discount rate $\delta_2^* = \max\{\delta_2, 0\}$ (23)

Example 1

To validate the theory, the numerical parameters are as follows:

T=6 months, t_1=5.5 months, c_p=500/unit, K=2000/unit, Δ =0.3 h=\$2/unit, c_o=\$300/cycle, F=2100, and G=2.

The problem can be formulated as follows:

Maximize:

$$TPU(\delta_1, \delta_2) = -375000\delta_1^2 - 375000\delta_2^2 - 375000\delta_1\delta_2 + 601625\delta_1 + 602875\delta_2 - 4850 \quad (24)$$

Subject to: $0 \le \delta_1 \le \delta_2 \le 1,\ 0 \le \delta_1 \le 0.3.$ (25)

Firstly, we consider the interior of the domain, $0 < \delta_1 < \delta_2 < 1,\ 0 < \delta_1 < 0.3.$ Using Eq. (22) and Eq.(23), the solution is δ_1 =0.534, δ_2 =0.537. However, this solution does not satisfy the constraint (25). Secondly, we consider the boundary of the domain: (a) $\{\delta_1 = 0,\ 0 \le \delta_2 \le 1\}$, (b) $\{\delta_1 = \delta_2,\ 0 \le \delta_2 \le 0.3\}$, (c) $\{\delta_1 = 0.3,\ 0.3 \le \delta_2 \le 1\}$, (d) $\{\delta_2 = 1,\ 0 \le \delta_1 \le 0.3\}$. In (a), the maximum is TPU(0,0.804)=\$237456; in (b), the maximum is TPU(0.3,0.3)= \$255250; in (c), the maximum is TPU(0.3,0.654)= \$302199, and in (d), the maximum is TPU(0.3,1)= \$257262. Hence, by comparison, the optimal profit per year is TPU(0.3,0.654)= \$302199, that is, the lower discount rate, δ_1 =0.3, the higher discount rate, δ_2 =0.654 and the optimal ordering quantity is Q*=2654 units. When without discount, the net profit per year is TPU(0.3,1)=\$257262. The profit increase is (302199/ 257262)-1=17.5%.

Example 2

The numerical parameters are the same as Example 1 except Δ =0.6. The problem can be formulated as follows:

Maximize:

$$TPU(\delta_1, \delta_2) = -375000\delta_1^2 - 375000\delta_2^2 - 375000\delta_1\delta_2 + 601625\delta_1 + 602875\delta_2 - 4850 \quad (26)$$

Subject to: $0 \le \delta_1 \le \delta_2 \le 1,\ 0 \le \delta_1 \le 0.6.$ (27)

Using Eq.(22) and Eq.(23), the interior solution of the domain is δ_1 =0.534, δ_2 =0.537, TPU(0.534, 0.537) =317556. In the boundary of the domain: (a) $\{\delta_1 = 0,\ 0 \le \delta_2 \le 1\}$, the maximum is TPU(0,0.804)=\$237456 ; (b) $\{\delta_1 = \delta_2,\ 0 \le \delta_2 \le 0.6\}$, the maximum is TPU(0.535, 0.535)=\$317555; (c) $\{\delta_1 = 0.6,\ 0.6 \le \delta_2 \le 1\}$, the maximum is TPU(0.6,0.6)=\$ 312850; (d) $\{\delta_2 = 1,\ 0 \le \delta_1 \le 0.6\}$, the maximum is TPU(0.302,1)=\$257264. Hence, by comparison, the optimal profit per year is TPU(0.534, 0.537) =317556, the lower discount rate, δ_1 =0.534, the higher discount rate, δ_2 =0.537 and the optimal ordering quantity is Q*=2391 units. When without discount, the net profit per year is TPU(0.6,1)=\$224000. The profit increase is (317556/ 224000)-1= 41.8%.

4. CONCLUSION

Internet marketing is a form of marketing strategy which utilizes the internet to promote business. The retailer orders a batch from the supplier and sells to customers with discounted selling price on the internet for promotion sake. Since the customers' demand depends on the selling price, therefore, how to determine the pricing strategy is most important to the retailer. In this study, the profit model is developed and the optimal solution of the model is derived. The sensitivity analysis shows that as the discount rate for market price (Δ) increases, $TPU(\delta_1^*, \delta_2^*)$ and % profit change all increase. That is, higher market price leads to higher profit. This study will also help the business managers understand the nature of internet market pricing dynamic.

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