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THE PROBLEM OF CHOOSING BEST ALTERNATIVE PROJECT IN TERMS OF RISK

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Abstract. *The paper deals with the problem of risky alternative projects and their choice. The problem is that under conditions of uncertainty and risk the application of several methods does not have to give an exact answer to the question of who is the best alternative project. That tells us that the correct selection of the project depends not only on the selection of method for the evaluation, but also from the decision-maker, types of problems and his views on these issues.*
Key words: *uncertainty and risk, alternative project, methods of evaluation*

1. INTRODUCTION

Thirty years of experience in the application of dynamic methods for project evaluation has pointed to certain system weaknesses. The key weakness is that these methods are all based on fixed cash-flow projections, meaning that their implementation assumes or claims that future is certain.

To eliminate these weaknesses, "the third generation" of methods for financial evaluation of projects appeared in the late 1990s headed by the method of real options, Monte Carlo simulation, decision tree, and optimization methods.

The above methods emerged as a response to inadequacy of traditional methods for evaluating the projects in terms of uncertainty. The foundation for their implementation should be the creation of diverse scenarios and simulations of future effects and recognition of the fact that managerial flexibility has a value and such value must be included in the value of the project as a whole. The main advantage of new methods for evaluating the projects is a significant reduction of space for making mistakes in the projection of essential inputs and making a final investment decision [1].

If the analysis and evaluation of engineering investment projects does not take into account the risk and uncertainty, and if we assume that certain inputs are entered, then all of these methods in the end must show the same result. That means that if a project is acceptable, it will show each method applied. What makes them eventually different is the way how is shown this result. However, under conditions of uncertainty, it is not necessarily so.

The aim of this paper is to show that under condition of risk and uncertainty over the application of several methods does not have to give an exact answer to the question of who is the best alternative project.

2. USED METHODS

The terms risk and uncertainty in decision-making is usually defined as follows: Decision making under uncertainty occurs when the probability of those states of nature is not known or when the decision maker does not want to make use of the estimated probability distribution. If the likelihood of particular scenarios is known and significant for the decision maker, then we deal with decision making under risk [2].

In this paper, for the case to be examined, will be used first the method of expected net present value (with the assumed probability distribution), and then some of the methods that are used when the probability distribution is not known.

Expected NPV is the sum of the product of NPVs under different scenarios and their relevant probabilities (p). The following formula is used to calculate expected NPV:

$$E(NPV) = \sum \left(p_j\right)\left(NPV_j\right) - \textit{discrete case}\ .$$

The standard deviation is a probability-weighted deviation from the expected value. In practice, the calculation of the variance is easier when we use the following formula [3]:

$$V(NPV) = E\left(NPV^2\right) - \left[E\left(NPV\right)\right]^2$$

Standard deviation (σ) is the square root of the variance:

$$\sigma = \sqrt{V}$$

In decision theory, there are certain rules (methods) to make decisions when there are more alternative projects, but there is not an opportunity to assess the probability (or probability distribution) to enactment certain events. When we have more mutually exclusive alternatives, and, given the uncertain future, we are unable to predict the unambiguous value of a method of decision-making (for example, NPV) for each alternative, suitably is this problem to present in the form of matrix decision [4].

Suppose that in a decision-making process is available m alternative projects $(A_1, A_2, A_3, ..., A_m)$, which can produce n states in the future $(S_1, S_2, S_3, ..., S_n)$. Now we can form the following matrix:

Table 1. Matrix decision

	S_1	S_2	S_3	.....	S_n
A_1	s_{11}	s_{12}	s_{13}	.....	s_{1n}
A_2	s_{21}	s_{22}	s_{23}	.....	s_{2n}
A_3	s_{31}	s_{32}	s_{33}	.....	s_{3n}
..	.....				
..	.....				
..	.....				
A_m	s_{m1}	s_{m2}	s_{m3}	..	s_{mn}

In this matrix $s_{11}, s_{12}, s_{13}, \cdots\cdots, s_{mn}$ are elements of the decision matrix, which represent the values of the adopted method of decision-making (for example, NPV).

The most well-known methods (rules or criteria) for the decision making under uncertainty are the next five:

- Wald's Maximin rule
- Maximax rule
- Hurwicz's rule
- Savage-Niehans rule
- Laplace's rule.

Maximin rule or Wald's rule (according to its founder A. Wald) is a rule that is based on the pessimistic prediction of results. Wald assumes that, if the future states are unknown, we should take the most cautious attitude and thus developed a theory of evaluation based on Maximin criteria. Maximin rule selects the alternative that has the most favorable result from the most unfavorable possible future state of alternative.

The Maximax criterion is an optimistic approach. It suggests that the decision maker examine the maximum payoffs of alternatives and choose the alternative whose outcome is the best. This criterion appeals to the adventurous decision maker who is attracted by high payoffs. This approach may also appeal to a decision maker who likes to gamble and who is in the position to withstand any losses without substantial inconvenience. [5]

Hurwitz's rule combines two previous extreme rules. The selected alternative has a higher score than alternative selected maximin rule and smaller than alternative selected maximax rule. The compromise is achieved by the introduction of so-called "optimistic parameter" $\lambda\,(0 \le \lambda \le 1)$, which reflects the awareness of the risk of decision-maker. The problem with this rule is to adopt the optimistic parameter λ (usually based on subjective opinions, or is determined by experiment). In the literature λ also referred to as the coefficient of pessimism [6].

Once we determine the parameter λ, then, according to Hurwitz's rule, for each alternative we calculate value from the following equation:

$$H_j = \lambda \cdot s_{ji_{\max}} + (1-\lambda) \cdot s_{ji_{\min}}$$

where $s_{ji_{\max}}$ is the maximum value and $s_{ji_{\min}}$ is the minimum value of alternative A_j For the best alternative is chosen it with maximum H_j.

It is obvious that with increasing values of λ, this rule becomes more optimistic. So for λ = 1 we have practically optimistic maximax rule and, conversely, for λ = 0 we have a pessimistic maximin rule.

Savage-Niehans rule minimizes maximum possible losses caused by erroneous or inadequate assessment. For this purpose we form a new matrix - risk matrix. The elements of this matrix we obtain by subtracting the individual values of certain columns of matrix decision from the maximum values in this column.

In the newly created risk matrix is allocated the largest value for each alternative - the maximum risk, and then we choose one alternative that has the lowest value among the maximum values. This rule has, in addition to certain benefits, the lack of which looks only at the extreme values of the columns.

Laplace rule assumes that, if it is not known the probability distribution of future events (states), it can be considered that these conditions are equally probable. Thus, each state of alternatives has a probability of 1/n (for n states), and then we choose the alternative whose average score has a maximum value.

The good side of this rule is that it takes into account all the conditions of one alternative, and not just extreme. On the other hand, the poor quality of this rule is that it assumes that all states in the future are equally probable and thus excludes the potential risk that the decision-maker accepted, expecting different

probabilities of certain states appearance in the future [4].

3. RESEARCH RESULTS

Suppose a company has prepared 4 alternatives of engineering investment projects. Data are given in the following table. Which alternative to choose?

Table 2. Data for four alternative projects

Altern.	NPV($)	p	NPV($)	p	NPV($)	p	NPV($)	p	NPV($)	p
A1	-40.000	0,2	10.000	0,2	60.000	0,2	110.000	0,2	160.000	0,2
A2	-60.000	0,1	0	0,2	80.000	0,4	100.000	0,2	180.000	0,1
A3	0	0,1	20.000	0,4	60.000	0,3	120.000	0,1	150.000	0.1
A4	-10.000	0,1	20.000	0,2	60.000	0,3	90.000	0,3	110.000	0,1

First, we apply the method of the expected NPV. We calculate the expected NPV and standard deviation for each of the four alternatives.

$$A_1: \; E(NPV)_1 = 60.000\,USD$$
$$V(NPV)_1 = 5.000 \cdot 10^6$$
$$\sigma_1 = 70.711\,USD$$
$$A_2: \; E(NPV)_2 = 64.000\,USD$$
$$V(NPV)_2 = 4.064 \cdot 10^6$$
$$\sigma_2 = 63.750\,USD$$
$$A_3: \; E(NPV)_3 = 53.000\,USD$$
$$V(NPV)_3 = 2.121 \cdot 10^6$$
$$\sigma_3 = 46.054\,USD$$
$$A_4: \; E(NPV)_4 = 59.000\,USD$$
$$V(NPV)_4 = 1.329 \cdot 10^6$$
$$\sigma_4 = 36.455\,USD$$

From these results we can conclude the following:
From the perspective of the expected present value we should choose alternative A2, as its expected present value is the largest. However, looking at the standard deviation as a criterion, we choose alternative A4, because it has the least deviation from the expected value (less deviation - lower uncertainty of the random variable).
Now we will apply the above-mentioned five rules in the same example, but assuming that the probability distributions are not known.
First we form a matrix decision and risk matrix:

Table 3. Matrix decision for four alternative projects

Alt.	S_1	S_2	S_3	S_4	S_5	Min. value	Max value
A_1	-40.000	10.000	60.000	110.000	160.000	-40.000	160.000
A_2	-60.000	0	80.000	100.000	180.000	-60.000	180.000
A_3	0	20.000	60.000	120.000	150.000	0	150.000
A_4	-10.000	20.000	60.000	90.000	110.000	-10.000	110.000
The maximum value in the column	0	20.000	80.000	120.000	180.000		

Table 4. Risk matrix

Alt.	S_1	S_2	S_3	S_4	S_5	Max risk
A_1	40.000	10.000	20.000	10.000	20.000	40.000
A_2	60.000	20.000	0	20.000	0	60.000
A_3	0	0	20.000	0	30.000	30.000
A_4	10.000	0	20.000	30.000	70.000	70.000

According Maximin rule, we seek the minimum value for each alternative. In our case, we see that the minimum values are: A_1= -40.000, A_2= -60.000, A_3= 0 and A_4 = -10.000, so we should choose A_3 alternative, allowing the decision maker at the same time prevented a utility value is negative.

Using Maximax rule, the choice of alternatives is among the four maximum values, which are: $A_1 = 160.000$, $A_2 = 180.000$, $A_3 = 150.000$ and $A_4 = 110.000$, so we should choose alternative A2.

According to Hurwicz's rule, selection of the best alternative depends on the parameter λ. In the investigated case, graphical representation of the expected value if we change λ from 0 to 1, is:

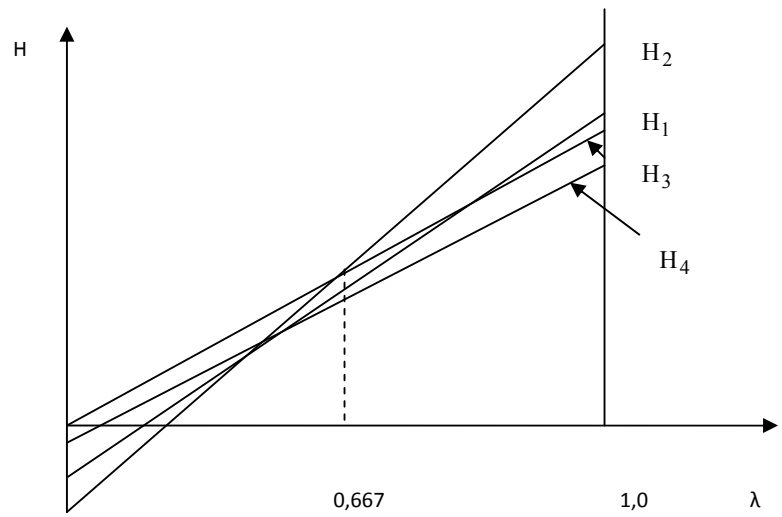

Figure 1. Graphical representation of the expected value

The Figure shows that the most favorable alternative is A_3 , if $0 \le \lambda \le 0{,}667$, and if $0{,}667 \le \lambda \le 1$, we need to choose alternative A_2.

Using Savage-Niehans rule, based on the risk matrix (Table 4), we should choose alternative A3, because it has the least amount of risk.

According to Laplace rule, for this case we obtain the following results for the five assumed conditions (as n = 5, the probability of occur any event is 0.2):

$$A_1 : (-40.000)\cdot 0{,}2 + 10.000\cdot 0{,}2 + 60.000\cdot 0{,}2 + \\ +110.000\cdot 0{,}2 + 160.000\cdot 0{,}2 = 60.000$$

$$A_2 : (-60.000)\cdot 0{,}2 + 0\cdot 0{,}2 + 80.000\cdot 0{,}2 + \\ +100.000\cdot 0{,}2 + 180.000\cdot 0{,}2 = 60.000$$

$$A_3 : 0\cdot 0{,}2 + 20.000\cdot 0{,}2 + 60.000\cdot 0{,}2 + 1 \\ +20.000\cdot 0{,}2 + 150.000\cdot 0{,}2 = 70.000$$

$$A_4 : (-10.000)\cdot 0{,}2 + 20.000\cdot 0{,}2 + 60.000\cdot 0{,}2 + \\ +90.000\cdot 0{,}2 + 110.000\cdot 0{,}2 = 54.000$$

Based on these results, we should choose alternative A_3 as the best, because its expected present value is the largest.

Summary of the results of this research gives the following table:

Table 5. Summary of the results

APPLIED METHOD	SELECTED ALTERNATIVE
1. a. Expected NPV 1 .b. Standard deviation	A2 A4
2. Maximin rule	A3
3. Maximax rule	A2
4. Hurwitz rule $0 \le \lambda \le 0{,}667$ $0{,}667 \le \lambda \le 1$	 A3 A2
5. Savage-Niehans rule	A3
6. Laplace rule	A3

4. CONCLUSION

As this paper has shown, for projects with a higher or lower degree of risk and uncertainty, different methods may give different results, which tells us that the correct selection of the project depends not only on the selection of method for the evaluation, but also from the decision-maker, types of problems and his views on these issues.

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