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METHOD FOR COLUMN CONSTRUCTION OF FULL FACTORIAL DESIGNS FOR FACTORS ON TREE LEVELS USING TAGUCHI'S ORTHOGONAL ARRAYS

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Abstract. *This paper present one of the methods for construction of columns for matrices of full factorial designs with factors on tree levels. Method is based on Taguchi's approach to construction of orthogonal arrays using Latin squares. Therefore it is applicable for both approaches when orthogonal arrays are viewed as a full factorial designs, rather than it's original form as a fractional factorial designs. This enable construct and use of matrices for full factorial designs or fractional factorial designs for tree level factors. This, further enables use of experimental matrices for full factorial designs with any number of factors.*

Key words: *open and closed full factorial designs, standard Latin squares, tree level factors, partition of interaction, characteristic rows*

1. INTRODUCTION

Theoretical basics for design of experiments, namely factorial designs are initiated on the beginning of the 20th century [1], while broad use, especially in industry are ignited by introduction of Taguchi's methods in late 1980-ties and 1990-ties [1]. Despite grate critique of Taguchi's approach to experimental design [1, 2, 3]

Theoretical reviews and development of experiments are slowed down and reoriented on books [4, 5]. On the other hand use of Taguchi's methods are wide in use in engineering and other research fields [6, 7, 8], with more frequent use of factors on three levels.

This paper presents one of the methods for construction of Taguchi's orthogonal arrays (OA) for tree level experiments, as well as full factorial designs for tree level factors. It is proven that full factorial designs for tree factors correspond to OA $L_i\left(3^j\right)$ [9].

Use of OA for Taguchi's method is standard procedure. In contrast to that, use of matrices (derived OA) in full factorial designs to a large degree facilitate non statisticians use of this experimental designs by avoiding Yates algorithm [1] or system of modules [10].

Construction of OA is based on Latin squares [11]. Latin squares are two dimensional matrices with letters [1] or numbers [10], i.e. *pxp* matrices (*p*=2,3,...), where every number appears only once in every column and in every row [1,10,12].

Elementary Latin squares are the standard or reduced Latin square. In standard Latin squares numbers constructing them are arranged in ascending order in first column and first row, with main diagonal from top right point to lower left point. Every standard Latin square generates set of $p!(p-1)!$ derived Latin squares [1].

2. METHODOLOGY

For tree level OA, Taguchi uses 3x3 Latin squares [11]. For 3x3 Latin squares there exists only one standard Latin square, with $3!(3-1)!=12$, with 11 of them are nonstandard [10]. For construction of Taguchi [11] uses standard Latin square, and one nonstandard, denoted as *p* and *q* (Table 1).

Table 1. Latin squares in use to construct 3^s OA [10]

	p				*q*	
1	2	3		1	3	2
2	3	1		2	1	3
3	1	2		3	2	1

Taguchi's OA were considered as a full factorial designs. Difference is in factor allocation. Taguchi's OA are named closed full factorial designs due difficulty to expand them for further research. Full factorial designs represented as OA-s are named

open designs, since they allow fold over, i.e. expansion of existing experiments if needed.
Further development of columns in matrices [13] is divided by factorial effects. Basic columns, i.e., columns contain main factorial effects have two degrees of freedom. Therefore interactions, depending of number of factors have more degrees of freedom and therefore occupy more than one column. Basically, they divided in certain number of partitions. Every partition occupy one column and have two degrees of freedom.

2.1. Construction of basic columns

For construction of factor levels basic columns of Taguchi's OA, from defined Latin squares 1, 2, 3, 1... results change of factor levels according the following sequence

$$\frac{3^k}{3}, \frac{3^k}{3^2}, \ldots, \frac{3^k}{3^{k-1}}, \frac{3^k}{3^k} \qquad (1)$$

For open factor levels in full factorial designs basic columns follow the sequence

$$\frac{3^k}{3^k}, \frac{3^k}{3^{k-1}}, \ldots, \frac{3^k}{3^2}, \frac{3^k}{3} \qquad (2)$$

2.2. Two factor interactions

Every two factor interaction have four degrees of freedom. Since every column in design have two degrees of freedom two factor interactions are divided on two partitions, that occupy those columns.
In order to construct factor levels for those columns, matching Latin squares from Table 1., can be expressed by columns (Table 2.)

Table 2. Adapted Latin squares for 3^k factorial design

p		
1	2	3
2	3	1
3	1	2
$\boldsymbol{p_1}$	$\boldsymbol{p_2}$	$\boldsymbol{p_3}$

q		
1	3	2
2	1	3
3	2	1
$\boldsymbol{q_1}$	$\boldsymbol{q_2}$	$\boldsymbol{q_3}$

Latin squares are divided by columns. Standard Latin square *p* becomes primary basic Latin square (Table 2), with elements representing columns

$$p_i = p_{(j)\downarrow}, i, j = 1,2,3 \qquad (3)$$

Other Latin squares in use - *q* is secondary basic Latin square (Table 2), with elements representing columns

$$q_i = q_{(j)\downarrow}, \; i = 1,2,3, \; j = 1,3,2 \qquad (4)$$

Depending on partition in column elements of primary or secondary Latin square are placed one above other.
Example of two factor interaction for smallest tree level OA (L_9) are presented at Table 3., where AB is partition I, while AB^2 is the partition II of interaction.

Table 3. Interaction columns for OA (L_9) with arrangement of parts of Latin squares p and q

AB	**AB^2**
1	1
2 p_1	2 q_1
3	3
2	3
3 p_2	1 q_2
1	2
3	2
1 p_3	3 q_3
2	1
p	***q***

In case of two factor interactions for first partition *p* is transformation for AB. For second partition of Latin squares *q* was used, and therefore it is transformation for AB^2. Two factor interactions have only one transformation. Larger interactions have more than one transformation *p* and *q*.

2.3. Three factor interactions

Tree factor interactions have eight degrees of freedom. They have four partitions, i.e. occupy 4 columns of factorial design. For three factor interactions it's necessary to determine two combination of transformations. Two consecutive transformations define mode of construction of partition of interaction column. For interaction ABC transformations for individual partitions are shown in Table 4.

Table 4. Method for construction of transformations of Latin squares elements for construction of partitions for tree factor interactions

ABC			**AB^2C**			**ABC^2**			**AB^2C^2**		
$p_1\ p_2\ p_3$		1 2 3	$q_1\ q_3\ q_2$		1 3 2	$p_1\ p_3\ p_2$		1 3 2	$q_1\ q_2\ q_3$		1 2 3
$p_2\ p_3\ p_1$	$\mapsto$	2 3 1	$q_2\ q_2\ q_3$	$\mapsto$	2 1 3	$p_2\ p_1\ p_3$	$\mapsto$	2 1 3	$q_2\ q_3\ q_2$	$\mapsto$	2 3 1
$p_3\ p_1\ p_2$		3 1 2	$q_3\ q_1\ q_1$		3 2 1	$p_3\ p_2\ p_1$		3 2 1	$q_3\ q_1\ q_1$		3 1 2
		$p_1\ p_2\ p_3$			$q_1\ q_2\ q_3$			$q_1\ q_2\ q_3$			$p_1\ p_2\ p_3$
p	$\mapsto$	***p***	***q***	$\mapsto$	***q***	***p***	$\mapsto$	***q***	***q***	$\mapsto$	***p***

2.4. Five factor interactions

Five factor interaction have 32 degrees of freedom, therefore 16 partitions is distributed in 16 columns in OA. Smallest OA for this design is L_{234}. Since large amount of factor levels in columns it was needed to reduce way of presentation. Let rows which represent begin of transformation element of Latin square be called *characteristic row*.

Characteristic rows are such that first transformation results from first and second characteristic rows (1 and 4). Second transformation is obtained from first transformation and characteristic row 3 (row 10 in OA), Third transformation is result of second transformation and characteristic row 4 (row 28 in OA) etc. Complete transformations are presented at Table 5.

Table 5. Characteristic rows for five factor interaction

Char.row	ABCDE	AB^2CDE	ABC^2DE	AB^2C2DE	$ABCD^2E$	AB^2CD^2E	ABC^2D^2E	$AB^2C^2D^2E$
1	(1)↓	(1)↓	(1)↓	(1)↓	(1)↓	(1)↓	(1)↓	(1)↓
4	(2)↓	(3)↓	(2)↓	(3)↓	(2)↓	(3)↓	(2)↓	(3)↓
10	(2)↓	(2)↓	(3)↓	(3)↓	(2)↓	(2)↓	(3)↓	(3)↓
28	(2)↓	(2)↓	(2)↓	(2)↓	(3)↓	(3)↓	(3)↓	(3)↓
82	(2)↓	(2)↓	(2)↓	(2)↓	(2)↓	(2)↓	(2)↓	(2)↓

Char.row	$ABCDE^2$	AB^2CDE^2	ABC^2DE^2	$AB^2C^2DE^2$	$ABCD^2E^2$	$AB^2CD^2E^2$	$ABC^2D^2E^2$	$AB^2C^2D^2E^2$
1	(1)↓	(1)↓	(1)↓	(1)↓	(1)↓	(1)↓	(1)↓	(1)↓
4	(2)↓	(3)↓	(2)↓	(3)↓	(2)↓	(3)↓	(2)↓	(3)↓
10	(2)↓	(2)↓	(3)↓	(3)↓	(2)↓	(2)↓	(3)↓	(3)↓
28	(2)↓	(2)↓	(2)↓	(2)↓	(3)↓	(3)↓	(3)↓	(3)↓
82	(3)↓	(3)↓	(3)↓	(3)↓	(3)↓	(3)↓	(3)↓	(3)↓

Based on Table 5, combinations of transformation of elements in used Latin squares are shown in Table 6.

Table 6. Transformations for five factor interaction

ABCDE	$p \circ p \circ p \circ p$	$ABCDE^2$	$p \circ p \circ p \circ q$
AB^2CDE	$q \circ q \circ p \circ p$	AB^2CDE^2	$q \circ q \circ p \circ q$
ABC^2DE	$p \circ q \circ q \circ p$	ABC^2DE^2	$p \circ q \circ q \circ q$
AB^2C^2DE	$q \circ p \circ q \circ p$	$AB^2C^2DE^2$	$q \circ p \circ q \circ q$
$ABCD^2E$	$p \circ p \circ q \circ q$	$ABCD^2E^2$	$p \circ p \circ q \circ p$
AB^2CD^2E	$q \circ q \circ q \circ q$	$AB^2CD^2E^2$	$q \circ q \circ q \circ p$
ABC^2D^2E	$p \circ q \circ p \circ q$	$ABC^2D^2E^2$	$p \circ q \circ p \circ p$
$AB^2C^2D^2E$	$q \circ p \circ p \circ q$	$AB^2C^2D^2E^2$	$q \circ p \circ p \circ p$

2.6. Pattern of transformation construction

For construction of combination of transformations following applies:

- For construction interaction with *i* factors there exists 2^{i-1} possible combinations of transformations
- Number of combinations in one transformation is *i*-1
- Pattern of transformation behavior could be defined according the rules which can be delivered on the basis of previous transformation as

$$\begin{aligned} &(p \wedge (2)\downarrow) \Rightarrow p \\ &(p \wedge (3)\downarrow) \Rightarrow q \\ &(q \wedge (2)\downarrow) \Rightarrow q \\ &(q \wedge (3)\downarrow) \Rightarrow p \end{aligned} \tag{5}$$

- Pattern also can be developed based on next transformation as

$$\begin{aligned} &(p \wedge (2)\downarrow) \Rightarrow p \\ &(p \wedge (3)\downarrow) \Rightarrow q \\ &(q \wedge (3)\downarrow) \Rightarrow p \\ &(q \wedge (2)\downarrow) \Rightarrow q \end{aligned} \tag{6}$$

3. CONCLUSIONS

Presented methodology is convenient for development of large orthogonal arrays for factors on tree lever either for Taguchi's or traditional approach for full factorial designs since:

- Simplify existing techniques by avoiding use of Yates algorithm or system of modules
- Unify experimental designs in matrix regardless of the number of factor levels
- Connects two similar techniques Taguchi's and traditional approach (closed and open designs)
- Enables construction of matrices with any number of factors larger than one
- Facilitate easier application in practice during the preparation, planning, executing, data collecting and analysis, especially for engineers that are only a users, with limited knowledge in statistics

Shortcomings of methodology is that is applicable only for prime numbers. Reason for this lies in selection of adequate standard Latin squares. Yates and system of modules is applicable only on prime numbers, therefore it is possible to obtain

appropriate standard Latin squares for construction of Orthogonal arrays. For composite numbers problem is selection of standard Latin squares for factorial designs as for example for Orthogonal arrays for four level factors [14].

REFERENCES

[1] Montgomery, DC (2001) *Design and Analysis of Experiments*, 5th Edition, John Willey & Sons, Inc, New York

[2] Box, G., Bisgaard, S., & Fung, C. (1988). An explanation and critique of Taguchi's contributions to quality engineering. *Quality and reliability engineering international*, *4*(2), 123-131.

[3] PIGNATIELLO JR, J. J., & RAMBERG, J. S. (1991). Top ten triumphs and tragedies of Genichi Taguchi. *Quality Engineering*, *4*(2), 211-225.

[4] Hedayat, A. S., Sloane, N. J. A., & Stufken, J. (2012). *Orthogonal arrays: theory and applications*. Springer Science & Business Media.

[5]Taguchi, G., Chowdhury, S., & Wu, Y. (2005). *Taguchi's quality engineering handbook*. Wiley.

[6] Hamidi, M., Azadi, A., Ashrafi, H., Rafiei, P., & Mohamadi-Samani, S. (2012). Taguchi orthogonal array design for the optimization of hydrogel nanoparticles for the intravenous delivery of small-molecule drugs. *Journal of Applied Polymer Science*, *126*(5), 1714-1724.

[7] Dixit, A. K., & Awasthi, R. (2015). EDM Process Parameters Optimization for Al-TiO2 Nano Composite. *International Journal of Materials Forming and Machining Processes (IJMFMP)*, *2*(2), 17-30.

[8] Pontes, F. J., de Paiva, A. P., Balestrassi, P. P., Ferreira, J. R., & da Silva, M. B. (2012). Optimization of Radial Basis Function neural network employed for prediction of surface roughness in hard turning process using Taguchi's orthogonal arrays. *Expert Systems with Applications*, *39*(9), 7776-7787.

[9] Veljković, ZA, Radojević S (2004) Using Taguchi's orthogonal arrays as a 3 level full factorial designs, *4th Annual ENBIS Conference*, Kopenhagen, Denmark, CD

[10] Federer, WT (1974) *Experimental Design, Theory and Application*, Oxford&IBH Publishing Co., 2nd reprint

[11] Taguchi, G (1991) *System of Experimental Design, Vol 1&2.,* Quality Resoruces, Kraus, New York, American Supplier Institute, Michigan

[12] Fisher RA, Yates, F (1934) The 6x6 Latin squares *Proceedings of the Cambridge Philosophical Society* **30**, pp. 492-507

[13] Veljković, ZA (2005) *Research on Transformations of Taguchi's Orthogonal Arrays for Application in Traditional Factorial Designs*, PHD thesis, University of Belgrade, Faculty of Mechanical engineering, Belgrade, Serbia (in Serbian)

[14] Veljković, Z. A., & Radojević, S., (2012). A Note on Four Level Taguchi's OA With Role of Latin Squares for Their Construction., *Proceedings of the 4th International symposium of industrial engineering SIE2012*, 109-112, Belgrade, Serbia.