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SIMULATIONS IN REANALYSIS PROCEDURES

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Abstract. *Most numerical simulations of physical systems are rife with sources of uncertainty. Uncertainty in simulations stems from the stochastic nature of geometric and physical parameters, indeterminate nature of initial/boundary conditions, and inadequacy of physical models coupled with discretization errors. The present will discuss the introducing of a probabilistic treatment of important problem parameters.*

Key words: *reanalysis, eigenvalues, design variables, simulation*

1.INTRODUCTION

Dynamic response of mechanical systems depends on structural parameters. The objective is to evaluate the structural response for successive modifications in the design avoiding the difficult solution of the modified equations. The structural modifications may be caused by external factors or by the designer in order to improve the characteristic of the response (eigenvalues and eigenvectors). Modification of dynamic characteristics means change of corresponding design variables to get desired dynamic behavior of structure. The design variables depend on the type of optimization problem. In the design of structural components, such as stiffened panels and cylinders, the design parameters represent the spacing of the stiffeners, the size and shape of the stiffeners, and the thickness of the skin. The thickness of plates, cross-sectional areas of bars, areas, moments of inertia, and torsion constants of beams represent sizes of the elements. Joints and members could be eventually added or deleted during the design procedure so that the geometry of the structures may be modified. Reanalysis methods can include the next activities:

(a) Modification in the geometry with no further change in the number of degrees of freedom.

(b) Modification of design variables (mass, damping and stiffness).

(c) Increase or reduction of the number of DOFs by changing the supporting manner and addition or deletion of joints and members.

(d) Alteration of the kind of material on some places if modification is possible. The main purpose of dynamic reanalysis is to provide numerical procedures to evaluate the structural response after modifications of design variables.

Simulation in structures is a very important field of investigation, due to its influence on subjects such as structures reliability and model validation amongst others. Simulation in structures can be used where the uncertainties introduced by random forces were applied to the structure Lin (1969). Conducted research Stanojevicat.all (2013), Tadic (2012, 2011), Dragojlovic (2012), Milanovic (2008) gives retrospective in uncertainty assessment in different research area.

Pascual (2012) cited that he followed the study of the case where uncertainty is introduced by random variables or by random fields modeling material properties (e.g., Young's modulus, mass density, Poisson's ratio, damping coefficient) or geometric parameters.

Cacciola at all (2005) did research on the procedure for the dynamic reanalysis of linear systems subjected to deterministic or stochastic loads. The structural modifications may be imposed by external factors (e.g. design alterations for operational reasons, or discrepancies between the predicted and measured properties of the structures) or by the designer in order to improve the characteristic of the response (e.g. layout optimization). Joints and members could be eventually added or deleted during the design procedure so that the topology of the structures may be modified. Reanalysis techniques are commonly devoted to efficiently determine the structural response produced bythe following events:

1. modification in the geometry with no further change in the number of degrees of freedom (DOFs);
2. alteration of dynamic characteristics of structural components (mass, damping and stiffness);
3. variation of the number of DOFs due to addition or deletion of joints and members;
4. alteration of loads due to both modification of the original number and position of joints and for changing in the intensity of external excitations.

Cacciola at all (2005) stated that most reanalysis methods are not able to deal with the last two modifications, which are usually named topological modifications as they imply a change in the dimension of the system due to addition or deletion of DOFs.

Kirsch and Liu (1997) focused a static reanalysis method by researching the characteristic of a modified initial design for the case of layout modification (no changes in the number of degrees of freedom).

Lecomte (2013) investigated the response of uncertain vibro-acustic and structural dynamic systems. In this paper, it is shown the comparison of the exact means, variances, covariances, as well as the exact stochastic and covariance coefficients, with their estimates obtained through Monte-Carlo simulations that confirmed the advantages of the analytical approach.

Voormeeren (2010) dealt with the problem of small random errors in substructure measurements in experimental dynamic substructuring using the frequency response functions (FRF). An uncertainty propagation method is derived, which allows the quantification of the uncertainty of the coupled system's FRFs propagated from uncertainties in measured substructure FRFs. A numerical example was used to verify the proposed method; the verification was performed through comparison with a Monte Carlo simulation.

2. METHODOLOGY

The problem of dynamic modification of a construction with the goal of improving dynamic characteristics has been a worldwide challenge for many researchers in previous decades (Trisovic (2007), Trisovic at all (2010), Allaboudiat all (2013).

The methods thereby used are widely different, from strictly mathematical do entirely experimental. Dynamic response of a mechanical structure must be improved by either (i) load control, or (ii) change in dynamic characteristics of a structure. Loads are often the result of interaction of the structure and its environment, so they are not easily controlled. In that case, it is important to know that the dynamic response can be improved by redesigning (reanalyzing) the dynamical characteristics of the structure. Having this in mind, the application of the techniques of reanalysis in obtaining the desired conditions for FE model of mechanical structures has shown a rapid improvement in previous decades. There are numerous techniques that are applied in dynamic reanalysis of mechanical structures. One of them has been already mentioned, *sensitivity analysis* that is successfully applied in general as well as in specific dynamical problems. The success of the procedure of dynamical modification depends on many factors, most important of which are: complexity of a structure including the boundary conditions, and modification method that a research team will choose to apply.

Although many papers have been published in the area of dynamic modifications of structures, the methodology of modification (reanalysis) of structures is still under intense development. In this paper, a procedure for dynamical modification that can be successfully applied to all types of structures is presented. Dynamic reanalysis is most often used in real structures that have poor dynamic behavior in exploitation. Successful "reparations" require a proper dynamic analysis and behavior diagnostics of observed structure. Application of results obtained by construction reanalysis achieves, among other results, prolongation of the life cycle of a construction.

3.1. Dynamic analysis and diagnostics of a model and its groups

Dynamic analysis and diagnosis of a model implies the analysis and interpretation of model behavior and its modification. On the basis of the analysis of energy distributions in main oscillation modes for all construction elements, the following cases are observed, on the grounds of which it is possible to derive the algorithm for reanalysis of similar structures.

I Elements in which the kinetic and potential energies (and the difference in their increase) are negligible with respect to other elements.
II Elements in which the kinetic energy is dominant compared to potential energy
III Elements in which the potential energy is dominant compared to kinetic energy
IV Elements in which the potential and kinetic energy exist and are not negligible in comparison with other elements

In simulation, a sequence of random numbers is generated according to the assumed distribution. Assume that the distribution of events occurrence, in our case, has the normal (Gaussian) distribution:

A random variable X with a probability distribution $f:\mathbb{R}\to\mathbb{R}>0,\ x\mapsto f(x)$

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

has normal distribution with the parameters μ , σ, which is written as $X\sim \mathrm{N}\left(\mu,\sigma^2\right) or \left(\mu,\sigma^2\right)$,

where μ is mathematical expectation and σ is standard deviation.

Probability distribution function of normal distribution is given by the expression:

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dt$$

3. EXAMPLE

Using the example of a cantilever beam, the application of the reanalysis steps has been demonstrated in determining the zones of the construction that are most sensitive to changes. Two models are observed: original and arbitrarily modified. The condition is that the modification be small. It is thus possible to considerably save calculation time, and it will be particularly demonstrated that by the line finite elements the reanalysis formula generates entirely reliable results. The type of modification is determined by the type of finite elements, type of boundary conditions, model geometry, and the like.

Consider a cantilever beam of length 1 m, rectangular cross-section, $b \times h = 100mm \times 50mm$, divided into 5 finite elements (Fig.1).

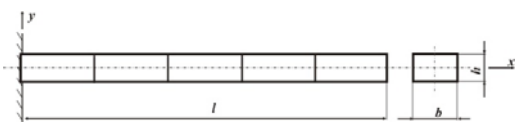

Fig. 1 Original cantilever beam
ρ=7833kg/m^3, E=206840000000 N/m^2;
b=0.1m, h=0.05m, l=1m

This cantilever beam is referred to as the original cantilever beam. For the analysis of sensitivity to changes, the *original cantilever beam* is modified across the entire length, with small modifications[1]. That cantilever beam is called a *modified cantilever beam* (Fig. 2).

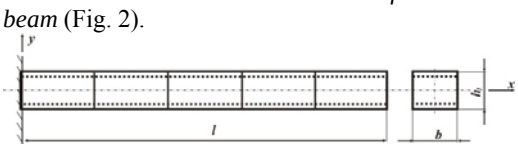

Fig. 2 Arbitrarily modifiedcantilever beam
b_1=b, h_1=1.1h

In this case, the chosen construction variable is the height of the rectangular cross-section *h*. Calculations are performed with the software package MatLab that possesses the function for calculating eigenvalues and eigenvectors. The lowest frequencies are always of the utmost interest for analysis. The table below (table 1) showstwo initial eigenvalues for the original cantilever beam and the modified one, where the height, as a construction variable, is increased by 10%.

Table 1

Original cantilever beam	Height increased by 10% across the entire length
Frequencies, f_{0i}[Hz]	Frequencies, f_{0i}[Hz]
260.24	286.26
41.51	45.66

Fig. 3 shows a digram of potential, ΔE_p ,and kinetic, ΔE_k , energy growth rates and their difference ΔE_p - ΔE_k for the increased height across the entire beam length, by 10 %, for the first oscillation mode. The first frequency of a modified cantilever beam is f'_{01} = 45.66Hz.

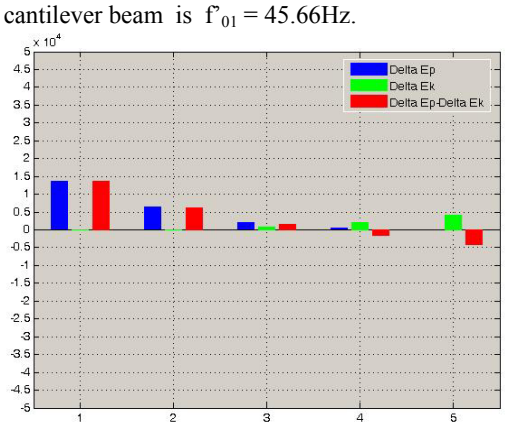

Fig. 3 Diagram of potential and kinetic energy growth rate distributions and their mutual difference for modified and original cantilever beam [J].

Fig. 5 displays a diagram of potential, ΔE_p ,and kinetic, ΔE_k , energy growth rates and their difference ΔE_p - ΔE_k for the modified cantilever beam after the first iterative step (Fig. 4) for the first oscillation mode.

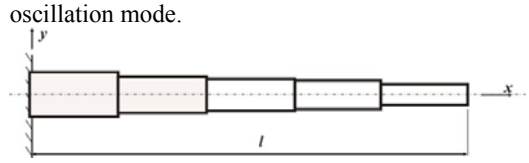

Fig. 4 Modified cantilever beam after the first iterative step

The aim of modification is to increase the frequency by 10 %. Note the convergence compared to the previous diagram, which is evidenced by reduced 'columns' characterizing the change in potential and kinetic energy growth rates.

[1] In theliterature dealing with dynamic reanalysis it is stressed thatmodifications should be small, so that the chosen modification process converges to the desired eigenvalues of the pairs, however it is not easy to determine what is 'small';

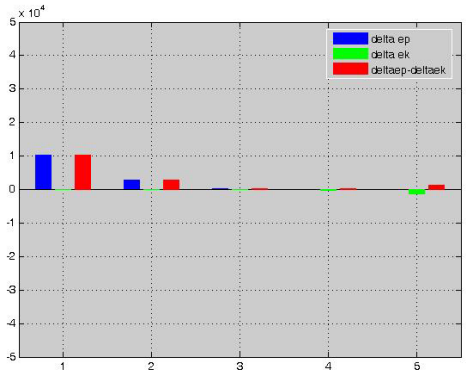

Fig. 5 Diagram of potential and kinetic energy growth rate distributions and their mutual difference for the modified cantilever beam after the first iterative step (Fig. 4,4), and the original cantilever beam [J].

Also, a significant conclusion related to the cantilever beam cross-section modification is that stiffness, i.e. cross-section height, should be increased in the fixed-point zone, while the beam's free end should be loosened, i.e. mass should be decreased in that zone.

4. SIMULATIONS

In simulation, a sequence of random numbers is generated according to the assumed distribution. Assume that the distribution of events occurrence, in our case, has the normal (Gaussian) distribution:

A random variable *X* with a probability distribution

$$f : \square \to \square > 0,\ x \mapsto f(x)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma} \right)^2 \right)$$

has normal distribution with the parameters μ , σ , which is written as $X \square \mathrm{N}\left(\mu,\sigma^2\right) or \left(\mu,\sigma^2\right)$, where μ is mathematical expectation and σ is standard deviation.

Probability distribution function of normal distribution is given by the expression:

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma} \right)^2 \right) dt$$

Further research included the execution of simulations *Ek, Ep*, growth rates *Ek* and *Ep*, differences in growth rates, first frequency in a cantilever beam and a modified beam for 1000 values of Young's modulus of elasticity according to the Gaussian distribution.

On the basis of simulation, the following results were obtained, as presented in the figures (Fig.11-16). The figures show the diagrams of distribution Ek, Ep, growth rates Ek and Ep, and differences in growth rates for each element separately.

Table2 shows the simulation results for the original beam frequency (frequency) and the modified beam frequency (frequency 1) for different values of Young's modulus of elasticity. Frequency growth rate was calculated as a difference between the modified beam frequency and the original beam frequency.

Tab. 2 Thesimulation results for the original beam frequency (frequency) and the modified beam frequency (frequency 1) for different values of Young's modulus of elasticity

No	Young's modulus	Frequency	Frequency 1
1	216008194057,00	43,52331847	47,87565032
2	221020777903,00	40,47158724	44,51874596
3	209951519584,00	41,8644712	46,05091832
4	213280508641,00	38,98414442	42,88255887
5	204137145899,00	39,82625243	43,80887767
6	218644848354,00	39,47510831	43,42261914
7	231827190259,00	40,42324048	44,46556453
8	222337773090,00	42,70373323	46,97410656
...			
1000	197682543995,00	39,15874433	43,07461876

A set of 1000 results obtained by the normal distribution (program **R** [2]) was used for further analysis where the Laplace criterion was applied to determine the uncertainty.

The Laplace criterion assumes equal probability for certain states to take place, so that probability represents

$$v\left(s_{ij}\right) = \frac{1}{m}$$

where*m* is the number of likely states (1000 in this case). Therefore the expected value is:

$$\overline{p_i} = \sum_{j=1}^{m} p_{ij} * v\left(s_{ij}\right) = \frac{1}{m}\sum_{j=1}^{m} p_{ij}$$

In Table 3 a division into the confidence intervals was performed and frequency of the observed quantities occurrence was calculated. The result for the occurrence of the oscillation frequency rounded growth rate indicates normal distribution.

Table 7. Confidence intervals

Frequency rounded growth rate	3,2	3,4	...	4,2	...	4,7
Frequency of occurrence	1	1	...	206	...	2

Fig. 6 shows normal distribution of the probability of the frequency of occurrence of eigenfrequencies growth rate difference. It is evident from the diagram that the highest probability of the

[2] **R** is a free software programming language and a software environment for statistical computing and graphics. The R language is widely used among statisticians and data miners for developing statistical software and data analysis. Polls and surveys of data miners are showing R's popularity has increased substantially in recent years.

occurrence of eigenfrequencies growth rate difference is 4.2, with the occurrence probability of 20.6%.

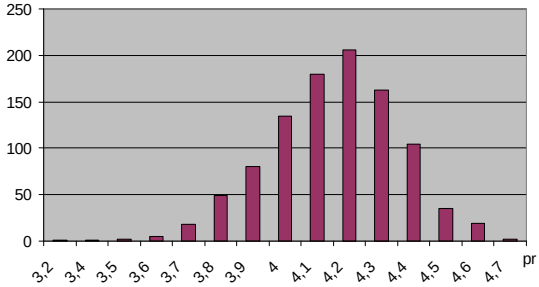

Fig 6 Normal distribution of occurrence probability of frequencies growth rate difference

By transforming the frequencies of occurrence of eigenvalues growth rate difference according to the Laplace criterion, we will obtain normal distribution of the occurrence probability, i.e. the degrees of uncertainty of the occurrence of eigenfrequencies growth rate difference.

The above text gives the probability of eigenfrequencies differences distribution in the original and the modified beam. Given that the normal distribution of eigenfrequencies growth rate difference was obtained, it is interesting to take a look at the probability distribution of eigenfrequencies in the original and the modified beam. Fig. 7, 8 and 9 present the distribution of eigenfrequencies occurrence probability for the original, the modified beam and optimized cantilever beam respectively.

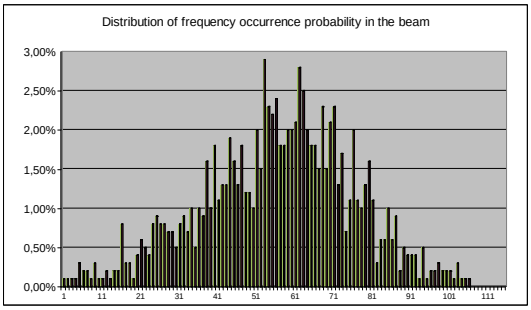

Fig.7. Distribution of frequency occurrence probability in the beam

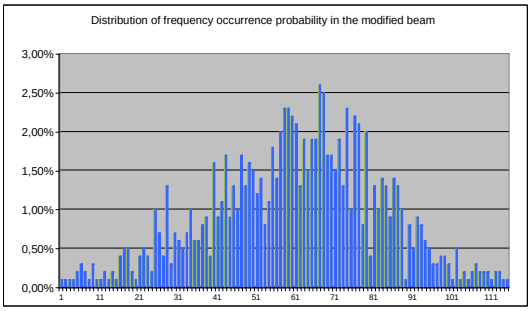

Fig. 8. Distribution of frequency occurrence probability in the modified beam

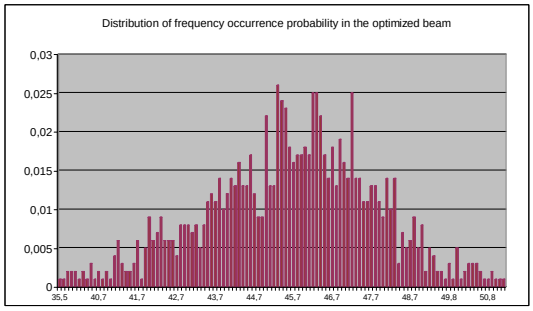

Fig. 9. Distribution of frequency occurrence probability in the optimized beam

5. CONCLUDING REMARKS

Studying the dynamic behavior of a construction can predict its response to change in shape, changes in size of its elements or change in materials used. Generally, the aim of system modification with respect to improvements in dynamic behavior is to increase eigenfrequencies and widen the distance between two neighboring frequencies. The specific importance lies in lowest frequencies and those close to the system exciting frequencies.

Assuming that Young's modulus of elasticity has normal distribution for 1000 simulation results, it is obtained that the frequency of free oscillations also has normal distribution in both the initial cantilever beam and modified beam and optimized beam. The results indicate that the behavior of the frequency of oscillations distribution correlates with the frequency of distribution of Young's modulus of elasticity and that the dependency exits irrespective of the cantilever beam design and shape respectively.

The analysis of uncertainty in the original, modified and optimized beams established for all three cases normal probability distribution in the rate of frequency occurrence. Difference was found in the interval of frequency normal distribution in the original cantilever beam compared to the distribution interval in modified and optimized beams. A broader confidence interval in modified and optimized beams indicates adverse effects of non-ideal material on the procedure of dynamic modification.

A versatile procedure for conducting reanalysis studies in the presence of uncertainty has been developed by. Combining Monte Carlo simulation tools with finite element modelling modules.

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REFERENCES

Allaboudi, E.,Maneski,T., Trisovic, N.,Ergić, T., „Improving structure dynamic behaviour using a reanalysis procedures technique“, Journal: Tehnički vjesnik, Vol.20, No.2, Str. 297 - 304 , April, 2013, ISSN 1330-3651.

[1] Canfield, R.A., High-Quality Approximation of Eigenvalues in Structural Optimization, AIAA Journal, Vol. 28, No. 6, 1990, pp. 1116-1122.

[2] Dragojlovic P., Misita M., Milanovic D.D., Tadic D., Kirin S., Risk management and multicriteria optimization of production program, RevistaMetalurgia International, 2012, Vol. 17, No. 6, pp. 35-39. ISSN 1582-2214

[3] Ki, I.K., Nonlinear Inverse Perturbation Method in Dynamic Redisgn, PhD, Thessis, Michigen University, USA, 1983.

[4] Kirsch U, Liu S. Structural reanalysis for general layout modifications. AIAA J 1997;35:382–8.

[5] Lecomte, C., Exact statistics of szstem with uncertainties: Am analytical theorz of rank-one stohastics dynamics systems, Journal of Sound and Vibration 332 (2013), 2750-2776.

[6] Lin YK. Probabilistic thoery of strcutural dynamics. NY, USA: McGraw-Hill Inc; 1967.

[7] Maneski T., KOMIPS software, Monograph computer modeling and structures calculation, Faculty of Mechanical Engineering, University of Belgrade, 1998 [ISBN 86-7083-319-0].

[8] Mateus, H.C., MotaSoares, C.M., and MotaSoares, C.A., Sensitivity Analysis and Optimal Design of Thin Laminated Composite Structures, Computers and Structures, Vol. 41, No. 3, 1991, pp. 501-508.

[9] Milanović D.D., Misita M, Decision support and management support systems, FME, Belgrade, 2008. ISBN 978-86-7083-642-6.

[10] Nair, B.P., Keane, A.J., and Langley, R.S., Improved First-Order Approximation of Eigenvalues and Eigenvectors, AIAA Journal, Vol. 36, No. 9, September 1998, pp. 1722-1727.

[11] Pascual, B. Adhikari S., Combined parametric–nonparametric uncertainty quantification using random matrix theory and polynomial chaos expansion, Computers and Structures 112–113 (2012) 364–379.

[12] Pierfrancesco C., Impollonia N., Muscolino G., A dynamic reanalysis technique for general structural modifications under deterministic or stochastic input, Computers and Structures 83 (2005) pp. 1076–1085.

[13] Rao, S.S., and Reddy, C.P., Optimum Design of Stiffened Conical Shells with Natural Frequency Constraints, Computers and Structures, Vol. 14, Nos.1-2, 1981, pp 103-110.Conference, 2001.

[14] Rao, S.S., and Reddy, C.P., Optimum Design of Stiffened Cylindrical Shells with Natural Frequency Constraints, Computers and Structures, Vol. 12, Aug. 1980, pp 211-219.

[15] Rao, V.R., Iyengar, N.G.R., and Rao, S.S., Optimization of Wing Structures to Satisfy Strength and Frequency Requirements, Computers and Structures, Vol. 10, No. 4, 1979, pp. 669-674.

[16] Sedaghati, R., Suleman, A., and Tabarrok, B., Structural Optimization with Frequency Constraints Using the Finite Element Force Method, AIAA Journal, Vol. 40, No. 2, 2002, pp. 382-388.

[17] Sergeyev, O., and Mroz., Z., Sensitivity Analysis and Optimal Design of 3D Frame Structures for Stress and Frequency Constraints, Computers and Structures, Vol. 75, No. 2, 2000. pp. 167-185

[18] Stanojevic P., Orlic B., Misita Mirjana, Tatalovic N., Lenkey G., Online monitoring and assessment of emerging risk in conventional industrial plants: possible way to implement integrated risk management approach and KPI's Journal of risk research, (2013), vol. 16(3-4), pp. 501-512.

[19] Tadic D., Djapan M., Misita M., Stefanovic M., Milanovic D.D. A Fuzzy Model for Assessing Risk of Occupational Safety in Processing Industry, The International Journal of Occupational Safety and Ergonomics, 2012, Vol.18, No.2, 115-126.

[20] Tadic D., Milanovic D.D, Misita M., Tadic B., A new integrated approach to the problem of ranking and supplier selection under uncertainties, Proceedings of the Institution of Mechanical Engineers, Part B, Journal of Engineering Manufacture, London, UK, 2011. Vol. 225, No. B9, oo. 1713-1724, ISSN 0954-4089.

[21] Thomson, W.T., Theory of Vibration with Applications, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1972.

[22] Tong, W.H., Jiang, J.S., and Liu, G.R., Solution Existence of the Optimization Problem of Truss Structure with Frequency Constraints, Int. Journal oj Solids and Structures, Vpol. 37., No. 30, 2000, pp. 4043-4060

[23] Tong, W.H., and Liu, G.R., An Optimization Procedure for Truss Structure with Discrete Design Variables and Dynamic Constraints, Computers and Structures, Vol. 79, No. 2, 2001, pp. 155-162

[24] Trišović, N., Maneski, T., Kozak, D. „Developed procedure for dynamic reanalysis of structures“, Strojarstvo - Journal for Theory and Application in Mechanical Engineering, Vol. 52, Number. 2, pp: 147-158, 2010; ISSN 0562-1887.

[25] Trisovic, R., N., Modification of the Dynamics Characteristics in the Structural Dynamic Reanalysis, PhD, Thessis, University of Belgrade, Serbia, 2007

[26] Voormeeren S.N., Klerk d., Rixen D.J., Uncertainty quantification in experimental frequency based substructuring, Mechanical Systems and Signal Processing 24 (2010) pp. 106–118.

[27] Wang, B.P., and Pilkey, W. D., Eigenvalue Reanalysis of Locally Modified Structures Using a Generalized Rayleigh's Method, AIAA Journal, Vol. 24, No.6, 1986, pp. 983-990.

[28] Wang, D., Zhang, W.H., and Jiang, J.S., Truss Optimization on Shape and Sizing with frequency Constraints, AIAA Journal, Vol. 42, No. 3, 2004, pp. 622-630.

[29] Yoon, B.G. and Belegundu, A.D., Iterative Methods for Design Sensitivity Analysis, AIAA Journal, 26, November 1988., pp. 1413-1417.