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OPTIMAL SALE NUMBER OF KIND FOR SPECIALIZED SHOP WITH INCREASING DEMAND

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Abstract. *The marketing policy for the retailers is influenced by sale number of product kind. In this study, we consider selling more kind of product to promote business. Selling a variety of product is needed to offer larger warehouse, inventory and ordering costs. An algorithm is developed to derive a replenishment cycle and sale kind of item such that the total unit time profit is maximized. A numerical analysis is presented.*
Key words: *Marketing policy; Product kind; Warehouse*

1. INTRODUCTION

Development of marketing strategy is the way to survive in a competitive environment. Owing to business competition and capital limitation, many retailers are beginning to be trafficking in department stores into specialty products. For example, tire stores, battery shop, shoe shop, for sale the only one merchandise store. Another example, automotive peripheral products shop including tires, battery,…etc., pet supply store for trafficking of a variety of relative goods. In terms of consumer behavior, multi-commodity trafficking stores are more attractive to consumer. However, retailers selling a variety of goods need to offer larger warehouse, inventory and ordering costs (Each kind of commodity must be individually ordered). So faced with conflict of increasing demand and escalating costs, how to tradeoff in commodities is a very practical problem.

There has been much work regarding the marketing and inventory problem. By acquiring and analyzing information regarding customers and competitors, marketing can be viewed an external-focused functional area that determines "what" kind of products (or services) a company should provide through "which" channel at "what" price (Tang, 2010). Marketing through internet has become an indispensable pipeline, however, entity display still make customers feel relieved.

Perspectives for a theory of marketing are outlined. Both the so-called marketing and broadened marketing concepts are inadequate conceptualizations of the marketing process. A theory of marketing should be formulated at the macro rather than the micro level and emphasize the positive rather than the normative problem-solving dimensions of the subject matter (Arndt, 1980). The householder concept, including mastering the material world and the resulting spiritual growth, stresses the importance of action, also a criterion for success in business (Barnett, 1985). Chesbrough (2010) explored the barriers to business model innovation, which previous academic research has identified as including conflicts with existing assets and business models, as well as cognition in understanding these barriers. Since product display usually induces customers' desire to purchase, the retailer makes more order so that extra products can be displayed on the shelf. Teng et al. (2011) considered a two-echelon inventory system with returns and shortage backordering, and examined a situation when stock increases will result in more consumption. Chan et al. (2012) reflected the most recent advances on green industrial marketing, green/sustainable supply chains and their interplay in green industrial branding, and to explore future research directions. Peters et al.(2013) offered a platform for the exploration, comparison, application, and consideration of ontological choice and its implications in industrial marketing research. Dennis et al. (2014) investigated tacit knowledge exchange between sales and marketing and its ability to enhance marketing success (i.e., marketing program innovativeness, relative efficiency, and relative effectiveness). Royle and Laing specify (2014) specified any digital marketing skills gaps encountered by professionals working in

communication industries. They found that a lack of specific technical skills; a need for best practice guidance on evaluation metrics, and a lack of intelligent future proofing for dynamic technological change and development are skills gaps currently challenging the communication industry. However, none of the above researches consider marketing with displaying more kind of product. This study considers selling more number of product kind to promote business. The aim is to determine the optimal number of product kind and replenishment cycle so that the total unit time profit is maximized.

2. ASSUMPTIONS AND NOTATION

In this section, the following notation is used throughout this paper.

T	the replenishment cycle length
N	total kind of item
n	selling kind of item
r	unit profit per sold unit
$H(n)$	inventory holding cost per unit time including warehouse cost, which is function of selling kind;

$$H(n) = h(1 - b + \frac{bn}{N}), 0 \le b \le 1, \quad 1 \le n \le N.$$

Where h, b are constants, that means the unit inventory holding cost is increasing function of selling kind

C	ordering cost per replenishment cycle
c_o	ordering cost per kind of item per replenishment cycle
TC	total unit time cost
$T\pi$	total unit time profit

In developing the model, the following assumptions are made:

1. Customer's demand is assumed as the average of individual demand of item. The demand rate is deterministic, stationary, and uniform through time.
2. Customer's demand follows a deterministic function $D(n)$ of kind n such that:

$$D(n) = d(1 - a + \frac{an}{N}), 0 \le a \le 1, \quad 1 \le n \le N.$$

Where d, a are constants. This means that the customer's demand increases when the kind of item increase.

3 The replenishment is instantaneous.

4. No shortages are allowed.
5. The capacity of the warehouse is unlimited.

(4)

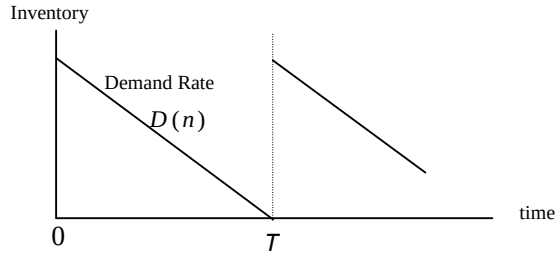

FIGURE 1. Customer's demand

3. MODELING AND ANALYSIS

In this section, we consider selling more kind of product to promote business. The supplier of multi-commodity trafficking store sells n kind of item. It is assumed that the total kind of item are N and the customer's demand follows a deterministic function $D(n)$ of kind n such that:

$$D(n) = d(1 - a + \frac{an}{N}), 0 \le a \le 1, \quad 1 \le n \le N,$$

where d, a are constants. This means that the customer's demand increases when the kind of item increase. While selling a variety of product is needed to offer larger warehouse, inventory, and ordering costs. This study tries to maximize the total unit time profit by determining (1) the replenishment cycle and (2) sale kind of item. A graphical representation of this inventory system is depicted in Figure 1.

If the total unit time cost is TC, then

$$TC(T,n) = \frac{1}{T}\text{[ordering cost + inventory holding cost]}$$

$$= \frac{1}{T}[C + nc_o + \frac{D(n)H(n)T^2}{2}]. \quad (1)$$

The total unit time profit is

$$T\pi(T,n) = \text{[unit time sales revenue- total unit time cost]}$$

$$= \frac{1}{T}[rD(n)T] - \frac{1}{T}[C + nc_o + \frac{D(n)H(n)T^2}{2}]. \quad (2)$$

Three cases are considered, that is, Case 1: when T is fixed and n is variable; Case 2: when n is fixed and T is variable, and Case 3: when both T and n are variables.

3.1 Case 1: when T is fixed and n is variable

Given that $T = T_o$, and denoting $T\pi_1(n) = T\pi(T_o, n)$, one has

$$\frac{d}{dn}T\pi_1(n) = \frac{rda}{N} - \frac{c_o}{T_o} - \frac{da(1 - b + \frac{bn}{N})T_o h}{2N} \quad (3)$$

(3)

$$\frac{d^2}{dn^2}T\pi_1(n) = \frac{-dabT_o h}{N^2} < 0. \quad (4)$$

That means $T\pi_1(n)$ is concave. By setting

$\frac{d}{dn}T\pi_1(n)=0.$ The optimal selling kind of item is as follows:

$$n^*=\frac{(2rdaT_o-2c_oN-adT_o{}^2h+2dabT_o{}^2h-bdT_o{}^2h)N}{2dabT_o{}^2h}. \quad (5)$$

3.2 Case 2: when n is fixed and T is variable

Given that $n=n_o$, and denoting $T\pi_2(T)=T\pi(T,n_o)$, one has

$$\frac{d}{dT}T\pi_2(T)=\frac{C+n_oc_o}{T}-\frac{dh(1-a+\frac{an_o}{N})(1-b+\frac{bn_o}{N})}{2} \quad (6)$$

$$\frac{d^2}{dT^2}T\pi_2(T)=\frac{-2(C+n_oc_o)}{T^3}<0. \quad (7)$$

That means $T\pi_2(T)$ is concave. By setting $\frac{d}{dT}T\pi_2(T)=0.$ The optimal replenishment cycle is as follows:

$$T^*=\sqrt{\frac{2(C+n_oc_o)}{dh(N^2-bN^2+bn_oN-aN^2+abN^2-2abn_oN+an_oN+abn_o{}^2)}}N. \quad (8)$$

3.3 Case 3: when both T and n are variables

$$\frac{\partial^2}{\partial T^2}T\pi(T,n)=\frac{-2(C+nc_o)}{T^3} \quad (9)$$

$$\frac{\partial^2}{\partial n^2}T\pi(T,n)=\frac{-dabTh}{N^2}. \quad (10)$$

$$\frac{\partial^2}{\partial T\partial n}T\pi(T,n)=\frac{2c_oN^2-dhaT^2N+2dhabT^2N-2dhabnT^2-dhbT^2N}{2N^2T^2}. \quad (11)$$

To show the concavity of $T\pi(T,n)$, the positivity is needed. Let $H(T,n)$ be the Hessian matrix of $T\pi(T,n)$:

$$H(T,n)=\frac{\partial^2}{\partial T^2}T\pi(T,n)*\frac{\partial^2}{\partial n^2}T\pi(T,n)-[\frac{\partial^2}{\partial T\partial n}T\pi(T,n)]^2. \quad (12)$$

From (9), (10), and (11), by using mathematical software MAPLE to calculate (11), it is too complicated to prove the positivity of $H(T,n)$. The illustrative example is to be presented to show the concavity of $T\pi(T,n)$ and derive the optimal solution of (T,n).

4. NUMERICAL EXAMPLE

The proposed model can be illustrated with the following numerical example.

Example 1. Let N=100, d=1000, r=2, h=2, C=80, c_o=1.5, a=1/3, b=1/2.

Case 1: When T is fixed and n is variable. Given that T=0.35, from (5), the optimal number of product kind is n^*=54, and $T\pi_1{}^*$=\$1005.

Case 2: When n is fixed and T is variable. Given that n=10, from (8), the optimal replenishment cycle is T^*=0.497 year, and $T\pi_2{}^*$=\$1018.

Case 3: When both T and n are variables (5)

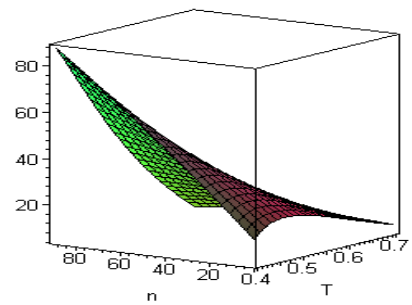

FIGURE 2. The graph of the Hessian matrix function of $T\pi(T,n)$ on [0.4, 0.7]×[1, 90].

Figure 2 (Using software Maple) shows the graph of the Hessian matrix function of $T\pi(T,n)$ on [0.4, 0.7]× [1, 90]. That means Hessian matrix of $T\pi(T,n)$ on [0.4, 0.7] × [1, 90] is positive. A graphical representation showing the concave function $T\pi$ is given in Figure 3. The optimal solution is derived by setting both $\frac{\partial}{\partial T}T\pi(T,n)=0$ and $\frac{\partial}{\partial n}T\pi(T,n)=0$. The optimal number of product kind is n^*=71, the optimal replenishment cycle is T^*=0.492 year. The optimal total unit time profit is $T\pi^*$=\$1048.

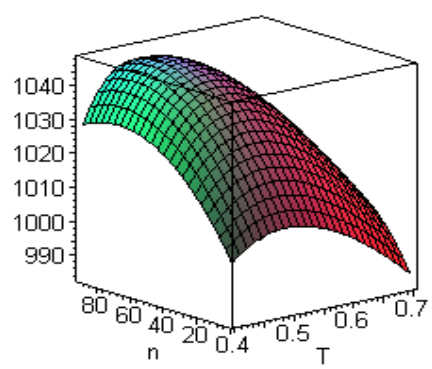

FIGURE 3. The graph of $T\pi(T,n)$ on [0.4, 0.7]×[1, 90].

5. CONCLUSION

This paper proposes and analyzes the optimal number of product kind and replenishment cycle so that the total unit time profit is maximized. The closed form of optimal solution is derived as another variable is fixed. The graphical illustration for concavity is presented when both T and n are

variables. Further research can be done to consider limited warehouse and capital.

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